Guidance for the Use of Significant Figures and Rounding Conventions in Water Quality Permitting

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Available from: http://www.deq.state.or.us/wq/pubs/imds/SigFigsIMD.pdf

1. Purpose

The purpose of this guidance is to explain the conventions for significant figures, rounding and precision for BWPC's Permitting Program. It represents the BWPC's current direction on the proper use of significant figures and rounding conventions in permit development. BWPC anticipates revising this document from time to time as conditions warrant.

2. Applicability

This document sets the conventions that BWPC water quality staff should use when developing permit limits and determining compliance with permit limits. Adherence to these conventions will ensure clarity and consistency in permit limit development and compliance determinations.

Because many of the permits issued by BWPC were developed prior to the development of this guidance, there will be instances in which permit writers and permit holders may agree to follow conventions established when those permits were written.

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3. Background

The process of developing and demonstrating compliance with water quality permits involves the analysis and interpretation of environmental data. This data is collected by a variety of public and private organizations employing a variety of sampling, analytic and data management practices that have varying levels of precision. The challenge for BWPC permit writers and compliance staff is how to interpret and use this data in a manner that is statistically relevant and consistent.

4. Conventions

There are three categories of conventions described in this document: significant figures, rounding and precision. In some cases, different conventions apply to measurements and calculations. Where there are differences, these are noted.

4.1 Significant Figures

Regardless of the measuring device, there is always some uncertainty in a measurement. Significant figures include all of the digits in a measurement that are known with certainty plus one more digit, which indicates the uncertainty of the measurement. For example, a mass reported as 1.1 g indicates the measurement is accurate to the nearest 0.1 g (i.e., the actual mass is between 1.0 and 1.2 g), but if the measurement is 1.10 g it is accurate to the nearest 0.01 g. This has implications both for permit limit development and for establishing compliance with a permit limit.

Table 1 below lists the conventions in use at BWPC regarding significant figures.

Table 1: Conventions for Significant Figures

Conventions	Example	No. of Significant Figures
1. All non-zero digits (1-9) are to be counted as significant.	23	2
	231	3
2. All zeros between non-zero digits are always significant.	4308	4
	40.05	4
3. For numbers that do not contain decimal points, the trailing zeros may or may not be significant. In this situation, the number of significant figures is ambiguous.	470,000	2 to 6
4. For numbers that do contain decimal points, the trailing zeros are significant.	0.360	3
	4.00	3
5. If a number is less than 1, zeros that follow the decimal point and are before a non-zero digit are not significant.	0.00253	3
	0.0670	3

As indicated in the third convention above, numbers that contain trailing zeros but that do not contain decimal points can be problematic. For example, “10” could be either one or two significant figures. There is no way to know what was intended unless there is a note that explicitly states how many significant figures there are.

Replacing “10” with “10.” is not a robust solution to this problem since Excel replaces “10.” with “10” and the information that the user intended to provide is lost.

The problem of how to interpret numbers with trailing zeros is pervasive enough that EPA changed the MCL for arsenic in drinking water from 10 ppb to 0.010 ppm to clarify the number of significant figures associated with the MCL.

4.2 Rounding

In reporting results and in calculating permit limits or mass loads, it is necessary to round the results to the correct number of significant figures. There are different rounding conventions in use, and BWPC has adopted a hybrid approach in which the rounding convention used for a number ending in 5 depends on the context. In reporting measured values, 5 is rounded to the nearest even number. For calculated values, 5 is rounded up. The conventions are listed in Table 2 below.

Table 2: Conventions for Rounding for Calculated and Measured Values

Conventions for Rounding	Examples	
	Rounding Off Calculated Values	**Rounding Off Measured Values**
1. If the digit being dropped is 1, 2, 3 or 4, leave the preceding number as-is.	1.11 → 1.1 1.12 → 1.1 1.13 → 1.1 1.14 → 1.1	Same
2. For calculations: If the digit being dropped is 5, round the preceding digit up.	1.15 → 1.2	N/A
3. For measurements: If the digit being dropped is 5, round the preceding digit to the nearest even number (0 is considered an even number when rounding).	N/A	1.25 → 1.2
4. If the digit being dropped is 6, 7, 8 or 9, increase the preceding digit by one.	1.16 → 1.2 1.17 → 1.2 1.18 → 1.2 1.19 → 1.2	Same

The conventions shown above for measured results are consistent with the rules for rounding found in Standard Methods for the Examination of Water and Wastewater, Part 1050 B.

For calculated results, rounding of 5 is consistent with the convention used by Microsoft Excel software.

Where commercial software packages and spreadsheets employ a different rounding routine (e.g., rounds up in all cases), the analyst should not change the results generated by the software.

If a permit holder chooses to use the same convention for calculated values as for measured values, the permit holder is allowed to do so, provided the permit holder is willing to commit to doing so on a consistent basis.
A shorthand version of the information presented in this section is as follows:

- **Measured values – the digit 5 should be rounded to the nearest even number.**
- **Calculated values– the digit 5 should be rounded up, unless the permit holder has chosen to follow the convention for measured values. The permit holder must do so on a consistent basis.**

4.3 Significant Figures, Decimal Places, and Reporting

There are two types of permit limits: those for which compliance will be determined based on the results of a laboratory or field measurement and those for which compliance will be based on the results of a calculation.

If compliance will be established based on a laboratory or field measurement, the number of significant figures in the permit limit should be the same as the number of significant figures associated with the laboratory or field measurement methodology.

If compliance will be determined based on the results of a calculation, the number of significant figures in the permit limit should be determined in a manner that is consistent with BWPC's rules for the determination of the number of figures to report as listed below.

Table 3: BWPC’s Conventions Determining the Number of Figures to Report

Convention	Example
1. **For addition or subtraction.** The number of decimal places* in the result is equal to the number of decimal places in the least precise value used in the calculation. *The number of decimal places is equal to the number of digits to the right of the decimal point.	13.681 – 0.5 = 13.181 becomes 13.2 0.5 is reported to only one decimal place so the final answer has one decimal place. Note that the number of digits in the answer is determined by the number of decimal places in the least precise measurement, and not by the number of significant figures.
2. **For multiplication or division**. The number of significant figures in the result is equal to the smallest number of significant figures of the values used in the calculation.	2.5 x 3.42 = 8.55 becomes 8.6 2.5 has the fewest significant figures (two) so the final result has two significant figures.
3. **When a calculation involves multiple arithmetic operations**. The number of significant figures is determined by both of the above rules with arithmetic operations performed in the following order: a. Operation(s) in parentheses b. Multiplication c. Division d. Addition e. Subtraction In a situation with multiple operations it is important not to round answers after each intermediate step. Instead keep track of the right most digit that would be retained based on rules 1 and 2 above (shown in the example on the right by an underline). The order of operations is seldom an issue in permitting. This information is included for completeness.	(2.5 x 3.42) + 13.681 – 0.5 = 22.731 becomes 22.7 **1) First do the operation in parenthesis** (in this case multiplication – Rule 2 above) = 8.55 + 13.681 – 0.5 **2) Next perform addition - Rule 1 above** = 22.231 – 0.5 **3) Then subtraction** – Rule 1 above = 22.731 all digits carried through = 22.7 final rounding In step 1, (based on rule 2), 8.55 would only be reported to two significant figures (retaining one decimal place). In this case, one place to the right of the decimal is the limiting digit for steps 2 and 3, and therefore the final result is reported to one decimal place.

4. **Values that are not considered**.

Values that are considered “exact” numbers are not included in the determination of the final number of significant figures. Here are some examples of exact values:

a. **Design flow** of a treatment facility.

   By contrast, the measured flow at a facility is not an exact number and does affect the number of significant figures in a calculation. Measured flows at treatment plants typically have two significant figures.

b. **Conversion factors.** Note: these should be selected so that the number of digits is at least that associated with measured values used in a calculation.

c. **Values below the Quantitation Limit.** Where the permittee is allowed to treat <QL as zero when averaging, the zero is not considered when determining the final number of significant figures.

d. **Counted values** such as:

   i. Bacteria measurements
   ii. The number of samples
   iii. Values denoting time (days, months, etc.)

Example 1:

For a POTW with a design flow of 1.5 MGD, the mass load of a pollutant measured at 5.25 mg/L is calculated as follows:

5.25 mg/L x 1.5 MGD flow x 8.34 = 219 lbs

The result contains three significant figures because the concentration of 5.25 contains three significant figures. The other numbers in the calculation, 1.5 MGD (design flow) and 8.34 (conversion factor), have no effect on the number of significant figures in the result.

Note that if the MGD of the facility were measured at the plant rather than being supplied by the design engineer, the number of significant figures associated with the flow would matter. Flow measurements typically have two significant figures.

Example 2:

What is the average of the following three concentrations: 4.6 mg/L, 2.3 mg/L and <QL

Answer: (4.6 + 2.3 + 0)/3 = 2.3 mg/L

The number of significant figures is equal to the number of significant figures for the detected concentrations.

The 3 in the denominator is a counted value and does not affect the number of significant figures or decimal places in the final rounding.

4.4 Permit Examples

Here are some examples of how these rules may apply when developing mass load limits or when determining compliance with mass load limits.

Example 1: Calculate a permit limit for the average daily mass load of ammonia.

Facility Info:

Average Dry Weather Design flow = 1.25 MGD
Average daily concentration of ammonia (measured as Total Ammonia as N) = 5.0 mg/L
Conversion factor from MGD and mg/L to pounds per day = 8.34

The allowable mass load for ammonia from this facility is calculated as follows:
1.25 MGD x 5.0 mg/L x 8.34 = 52.13 lbs/day → 52 lbs/day

Comments

The resulting permit limit has been rounded to two significant figures because that is how many significant figures are associated with the ammonia concentration. The number of significant figures in the permit limit is unaffected by the number of digits in the design flow or the conversion factor (see Conventions 4.a and b. in Table 3).

If the calculated result had been 52.5 lbs/day instead of 52.13 lbs/day, the permit limit would have been rounded up to 53 lbs/day (see Rounding Convention 2 in Table 2).

Note that if the allowable ammonia concentration was 10.0 or greater, the permit limit would contain three significant figures instead of two. This is because if the ammonia concentration is 10.0 or greater, it has three significant figures instead of two.

Example 2: Calculate the 7-day average concentration for ammonia.

Permit limit = 4.5 mg/L, sampled 4 times a week

Measured concentrations = 0.6, 2.5, 12.7 and <0.1 mg/L

(0.6 + 2.5 + 12.7 + 0)/4 = 3.95 mg/L → 4.0 mg/L

Comments

The result has been rounded to two significant figures because the permit limit contains two significant figures and it is rounded up (see Rounding Convention 3 in Table 3 and Rounding Convention 2 in Table 2).

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Note that the lab result 12.7 contains more significant figures than the permit limit. Lab results for ammonia have two significant figures when the values are below 10.0 mg/L and three when they are above 10.0 mg/L. This does not affect the final answer of 4.0 mg/L.

Note that the non-detect is treated as zero and it does not affect the number of significant figures in the final result (see convention 4.c. in Table 3). The value of 4 in the denominator also has no effect because it is a counted number (see Convention 4.d. in Table 3).

Example 3: Determine if the following facility is in compliance with their permit limit for average daily mass load of ammonia of 38 lbs/day.

Facility Info:

Average daily flow = 0.85 MGD
Average daily concentration of ammonia (measured as Total Ammonia as N) = 5.0 mg/L
Conversion factor from MGD and mg/L to pounds per day = 8.34

The mass load for ammonia from this facility is calculated as follows:
0.85 MGD x 5.0 mg/L x 8.34 = 35.5 lbs/day → 36 lbs/day

Comments

The result has been rounded off to two significant figures because that is how many significant figures are associated with the ammonia concentration and with the average daily flow from the facility. The conversion factor has no effect on the number of significant figures.